CHAPTER 1

INTRODUCTION

Introduction

The following notes are being developed for use in a post graduate course at McMaster University and possible future publication. These materials are the intellectual property of the author, Professor Robert T.H. Alden, and are not to be used for other than educational purposes by students registered in the course.

1.1 Scope

These notes begin with the assumption of a background that includes introductory material that is typically found in undergraduate courses in electric circuit analysis, electric machines, power and control systems, matrix methods, and problem solving using personal computers. The aim is to develop a competency in representing the various elements of modern power systems by computer based models, and using these models to predict system stability in the presence of small perturbations. The heart of system stability lies in the understanding of the basic system components, control systems, and their interactions. We begin by developing the equations for the synchronous generator in chapter 2, and the corresponding equivalent circuits in chapter 3. Simple models are then used to develop the approach in modeling a low order system in chapter 4, and performing eigen analysis in chapter 5.

More complex system representation is now presented. Techniques for assembling high order matrix models are developed in chapter 6. Exciter, stabilizer, governor, and turbine systems are developed in chapters 7 and 8. Complex shaft dynamics are introduced. Techniques for reducing high order matrix models are then treated in chapter 9. The concept of synchronizing and damping torque components is discussed in chapter 10 as an aid to extending our understanding of system behaviour and its control. The representation of induction motors and the use of an aggregate motor model to represent a group of motors is presented in chapter 11. The extension of these techniques to multimachine systems is introduced in chapter 12.

1.2 Conversion of Equations from SI to PU form

Equations are developed using the System Internationale (SI) standard for units of measurement. The use of Per Unit (PU) notation is widespread in both industrial and academic sectors, so the conversion from SI to PU will now be discussed. In so doing, the fundamental units of the generic equations will be clearly detailed. This may serve as both a review and a consolidation of previous knowledge. Note that time is kept in seconds and angles in radians.

1.2.1 Defining Per Unit System Base Quantities

Assuming balanced three-phase conditions and a wye equivalent circuit for impedances; we define the fundamental base quantities $S_b = S_{3\phi}$ and $V_b = V_{LN}$, usually the rated values, and then compute the remaining base values:

Thus $I_b = \dfrac{S_b}{3\ V_b}$ and $Z_b = \dfrac{V_b}{I_b}$. And since $\omega_b = \omega_o$, then $T_b = \dfrac{S_b}{\omega_o}$ and $\psi_b = \dfrac{V_b}{\omega_o}$

Any quantity in SI units is converted to PU value by dividing by its base value: $\rho_{PU} = \dfrac{\rho_{SI}}{\rho_b}$

1.2.2 Converting Flux Linkage Equations

A typical flux linkage equation is of the form:

$$\psi_f = L_f\ i_f - L_{md}\ i_d \quad \text{weber turns}$$

We multiply both sides by ω_0 and replace the inductance terms by reactances:

$$\omega_o\ \psi_f = X_f\ i_f - X_{md}\ i_d \quad \text{volts}$$

Then divide both sides by the voltage base, substitute for voltage as shown, and express in per unit notation:

$$\therefore \quad \frac{\omega_o\ \psi_f}{V_b} = \frac{X_f\ i_f}{V_b} - \frac{X_{md}\ i_d}{V_b}$$

$$\text{or} \quad \frac{\omega_o}{\omega_o}\ \frac{\psi_f}{\psi_b} = \frac{X_f}{Z_b}\ \frac{i_f}{I_b} - \frac{X_{md}}{Z_b}\ \frac{i_d}{I_b}$$

$$\text{or} \quad \psi_{fu} = X_{fu}\ i_{fu} - X_{mdu}\ i_{du}$$

Usually the u subscript is dropped if all equations are in PU form. Note that the SI and PU forms are identical except that inductance in SI is replaced by reactance in PU.

1.2.3 Converting Voltage Equations

A typical voltage equation is of the form:

$$v_d = - R_s \, i_d + \frac{d}{dt}\psi_d - \omega \, \psi_q$$

We divide both sides by the voltage base and substitute for voltage as shown:

$$\frac{v_d}{v_b} = - \frac{R_s}{Z_b} \frac{i_d}{I_b} + \frac{\frac{d}{dt}\psi_d}{V_b} - \frac{\omega}{\omega_o} \frac{\psi_q}{\psi_b} \qquad \text{per unit}$$

We note that:

$$\psi_d = \frac{d}{dt} (\psi_d) = \frac{d}{dt} (\psi_b \, \psi_{du}) = \psi_b \frac{d}{dt}\psi_{du} = \frac{V_b}{\omega_o} \frac{d}{dt}\psi_{du}$$

and substitute in the middle term on the right to obtain:

$$v_{du} = -R_{su} \, i_{du} + \frac{1}{\omega_o} \frac{d}{dt}\psi_{du} - \frac{\omega}{\omega_o} \psi_{qu}$$

Again, the u subscript is often dropped if all equations are in PU form.

1.2.4 Converting Torque and Power Equations

A typical torque equation is of the form:

$$T = 3 \, (\, i \, \psi + L \, i^2 \,) \qquad \text{N.m}$$

Multiplying both sides by ω_o to obtain a power equation, and dividing by $S_b = 3 \, V_b \, I_b$, we obtain:

$$\frac{T \, \omega_o}{3 \, V_b \, I_b} = [\, \frac{3 \, i \, \psi \, \omega_o}{3 \, I_b \, V_b} + \frac{3 \, X \, i^2}{3 \, V_b \, I_b} \,]$$

$$\text{Noting} \quad T_b = \frac{3 \, V_b \, I_b}{\omega_o} \, , \quad V_b = \psi_b \, \omega_o \, , \quad \text{and} \; Z_b = \frac{V_b}{I_b}$$

we substitute and obtain the per unit form of the torque equation:

$$T_u = i_u \, \psi_u + X_u \, i_u^2$$

1.3 Linearizing of Equations

In the same manner that equations are developed in SI units and then converted to PU form, the device and system representation implicitly includes non-linear effects which require adaptation in order to fit the linear form required for small disturbance analysis. These non-linearities result from material characteristics, control strategies, and interaction between variables. Material and control non-linearities are treated using non-linear mathematical functions. The interaction between variables results from multiplication or the use of trigonometric functions.

The field of dynamic analysis is sometimes viewed as small perturbation analysis only. Alternatively, it can be seen as encompassing the entire range from small perturbations where linearized techniques can be usefully employed, to large disturbances where numerical integration or Lyiapunov related techniques are required. It should be remembered that both aspects (small and large disturbances) are normally used in any practical investigation of system stability. The two aproaches are, in fact, highly complimentary.

In the following chapters, where we examine the application of eigen techniques (or modal analysis), we use linearized equations in the context of an appropriate operating point that reflects non-linear conditions. We will derive the linearized form of three kinds of equations; the straightforward case, one with variable products, and one with trigonometric functions.

1.3.1 Simple Linearization

Given an equation of the form:

$z = A\,x + B\,y + \ldots$

where A and B are non-linear constants, we establish the operating (or quiescent) point z_o, x_o, y_o and evaluate A_o and B_o. The linearized form of the equation is found by perturbing thus:

$z + \Delta z = A_o\,(x + \Delta x) + B_o\,(y + \Delta y) + \ldots$ at the point where $z = z_o$, $x = x_o$, $y = y_o$

$z_o + \Delta z = A_o\,(x_o + \Delta x) + B_o\,(y_o + \Delta y) + \ldots$

Subtracting out the quiescent component, the linearized incremental equation is:

$\Delta z = A_o\,\Delta x + B_o\,\Delta y + \ldots$

1.3.2 Linearization of Variable Products

If the equation is of the form (containing the product xy):

$z = A\,x\,y + \ldots$

where A is a non-linear constant, we establish the operating (or quiescent) point z_o, x_o, y_o and evaluate A_o. The linearized form of the equation is again found by perturbing thus:

$z + \Delta z = A_o\,(x + \Delta x)\,(y + \Delta y) + \ldots$ at the point where $z = z_o$, $x = x_o$, $y = y_o$

$z_o + \Delta z = A_o\,(x_o + \Delta x)\,(y_o + \Delta y) + \ldots$

$= A_o\,(x_o\,y_o + y_o\,\Delta x + x_o\,\Delta y + \Delta x\,\Delta y) + \ldots$

Subtracting out the quiescent component, the linearized incremental equation is:

$\Delta z = A_o\,(y_o\,\Delta x + x_o\,\Delta y + \Delta x\,\Delta y) + \ldots$

It is customary to neglect the "second order effect" term as it is usually negligible. Thus, the linearized form of the equation is:

$\Delta z = A_o\,(y_o\,\Delta x + x_o\,\Delta y) + \ldots$

1.3.2 Linearization of Trigonometric Functions

If the function is $\sin\delta$, then we perturb by $\Delta\delta$ and expand as follows:

$\sin(\delta_o + \Delta\delta) = \sin\delta_o\ \cos\Delta\delta + \cos\delta_o\ \sin\Delta\delta = \sin\delta_o + \cos\delta_o\ \Delta\delta$

where we have used the relations $\cos\Delta\delta = 1$ and $\sin\Delta\delta = \Delta\delta$ for small arguments. Subtracting out the quiescent component, the linearized incremental equation is:

$\Delta\sin\delta = \cos\delta_o\ \Delta\delta$

If the function is $\cos\delta$, then we perturb by $\Delta\delta$ and expand as follows:

$\cos(\delta_o + \Delta\delta) = \cos\delta_o\ \cos\Delta\delta - \sin\delta_o\ \sin\Delta\delta = \cos\delta_o - \sin\delta_o\ \Delta\delta$

where we have again used the relations $\cos\Delta\delta = 1$ and $\sin\Delta\delta = \Delta\delta$ for small arguments. Subtracting out the quiescent component, the linearized incremental equation is:

$\Delta\cos\delta = -\sin\delta_o\ \Delta\delta$